Mechanical vibration
(Introduction to vibration)

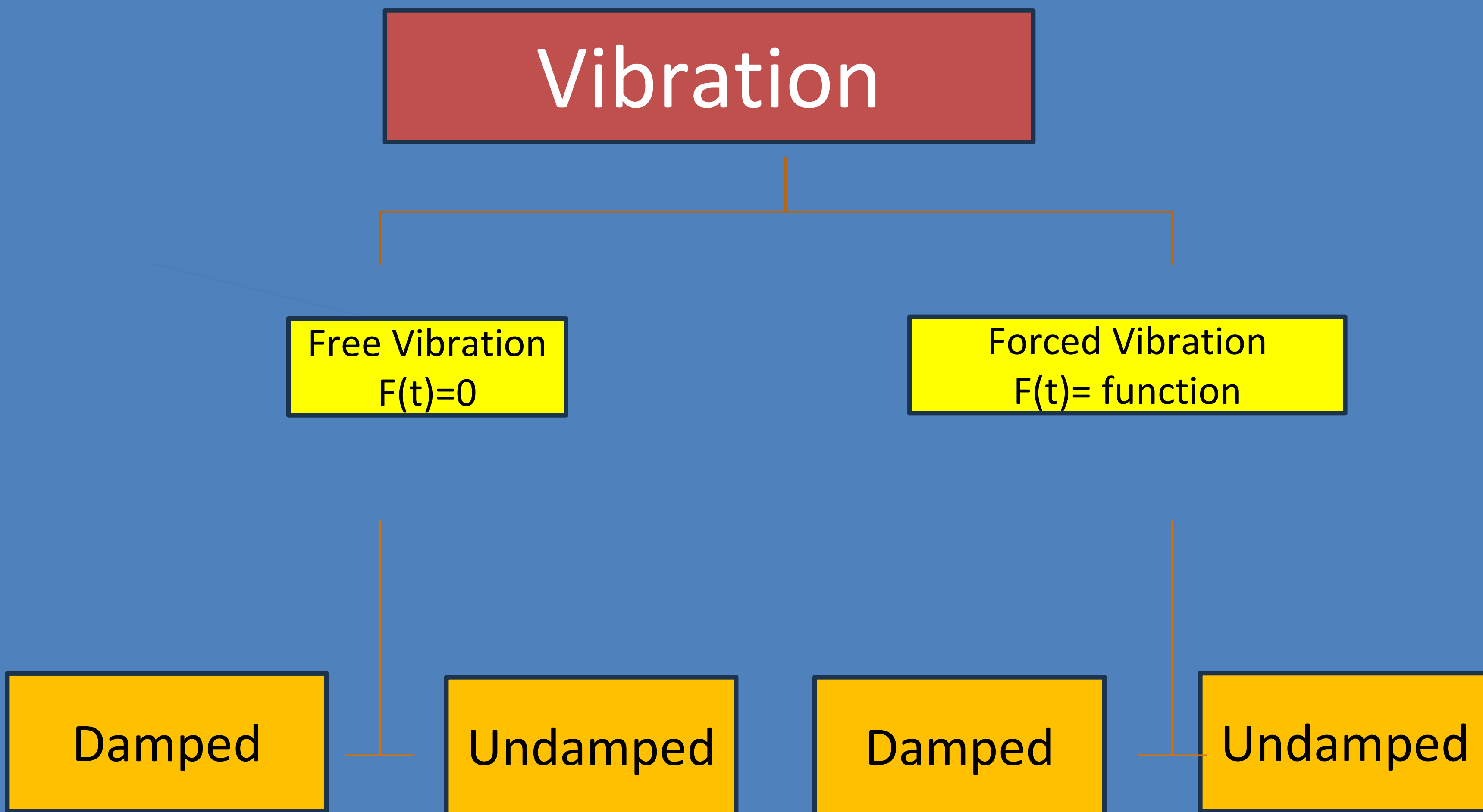
Faculty of Engineering
Mechanical Engineering Department

Dr. Rasha Afify

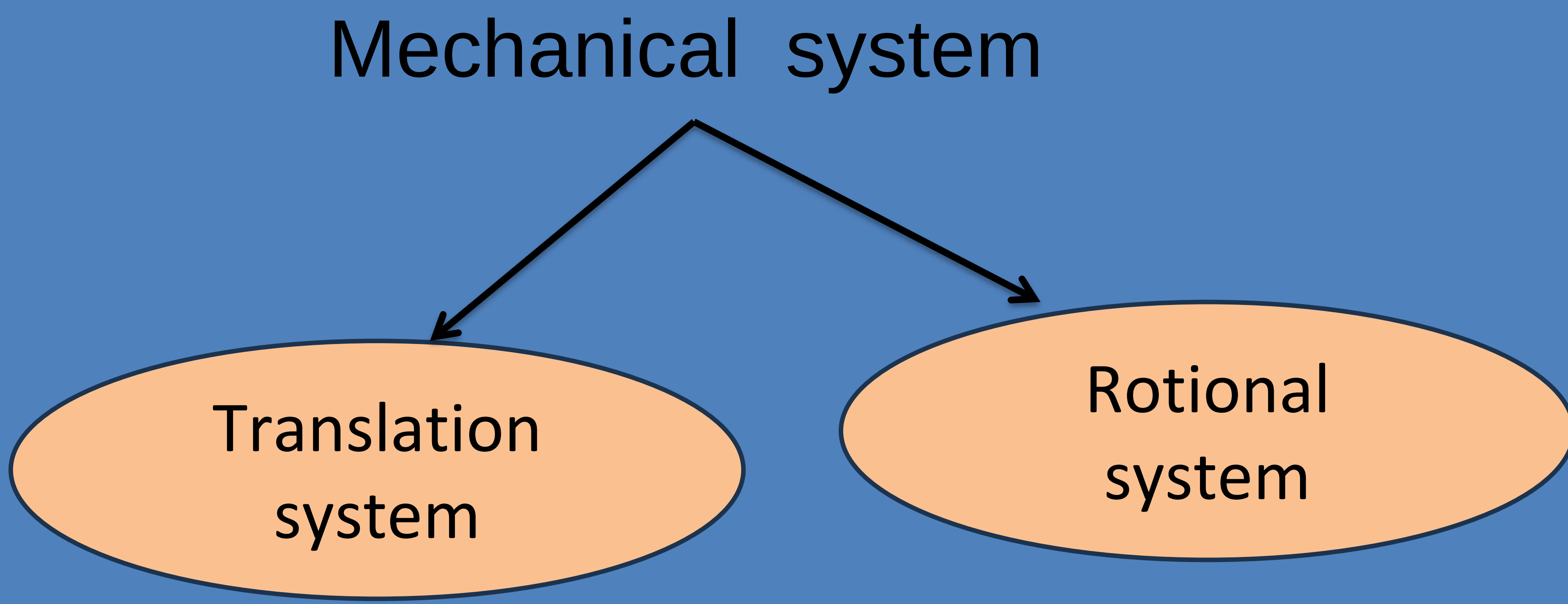
Objective

- Classification of vibration
- Basic components of mechanical system
  - Mass, spring and damper
- Equivalent springs (Combining stiffness)

Vibration is simply classified into



Mechanical System



Basic mechanical elements

- **The three basic elements of mechanical systems are:.**

  - Mass or disc (inertia element)
  - Spring
  - Viscous Damping element (damper)

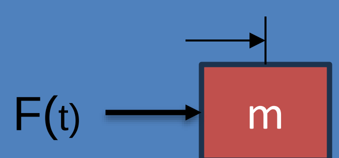
Translational mechanical elements

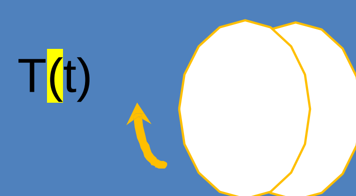
- **1- Mass Inertia**
  - Masses is an inertia element that store **K**inetic **E**nergy (KE)

Mass

Linear Mass | Rotational Mass

Translation | block | Rotational | Disck





F(t) = ma = mx

T(t) =JÖ

$$K.E= \frac{1}{2} m\dot{X}^2 = \frac{1}{2} m \dot{X}^2$$

$$K.E= \frac{1}{2} J\dot{\Theta}^2 = \frac{1}{2} J\,\omega^2$$

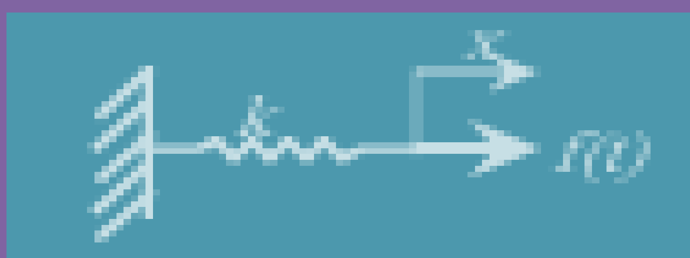
Translational mechanical elements

2. Spring: is a stiffness component that storing potential energy

Springs

- Linear Spring
- Rotating Spring

Spring force is linear and proportional to linear or angular displacement



$$f(t) = k\,x$$

$$P.E = \int f\,dx = \int k\,x\,dx = \frac{1}{2}k\,x^2$$

$$T(t) = k_t\,\theta$$

$$P.E = \int f\,d\theta = \int k_t\,\theta\,d\theta = \frac{1}{2}k_t\,\theta^2$$

Translational mechanical elements

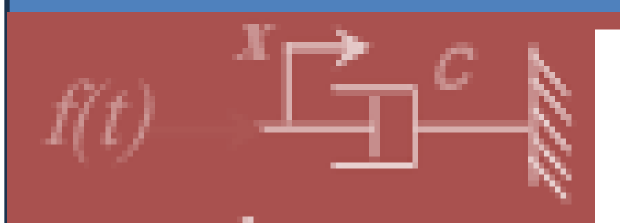
- **3- Viscous damping (friction) “damper”**
  - Damper is a dissipating component that Disperses

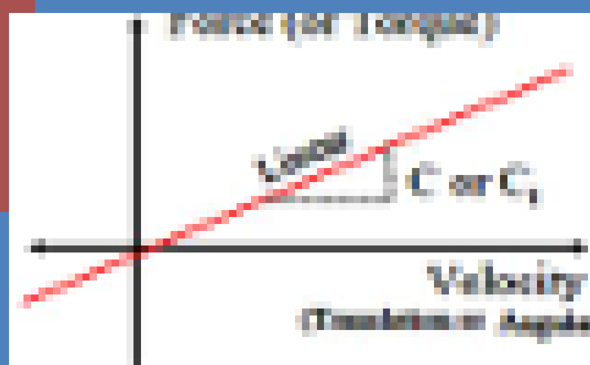
Damper

Linear Damper

Rotating Damper

Spring force is linear and proportional to linear or angular displacement





$$f_d = c\,\frac{dx}{dt} = c\,\dot{x} = c\,v(t)$$

$$D.E = \int f\,dx = \int c\,\dot{x}\,dx = \frac{1}{2}C\,\dot{x}^2$$

$$T_d = c_t\,\frac{d\theta}{dt} = c_t\,\dot{\theta} = c_t\,\omega(t)$$

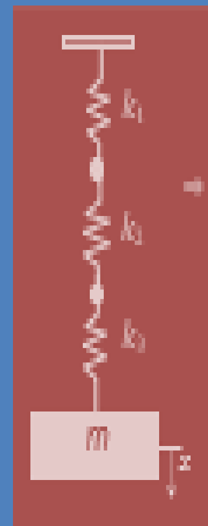
$$D.E = \int f\,d\theta = \int c_t\,\dot{\theta}\,d\theta = \frac{1}{2}C_t\,\dot{\theta}^2$$

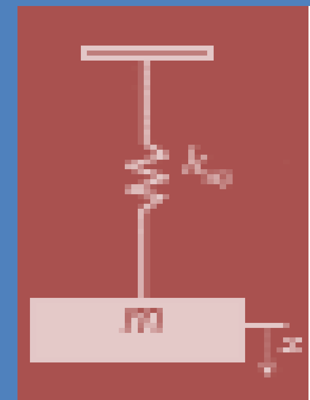
Equivalent springs (Combining stiffness)

▪Springs in series :

$x_{eq} = x_1 + x_2 + x_3 \quad \rightarrow (1)$

Since, $f(t) = k\,x$





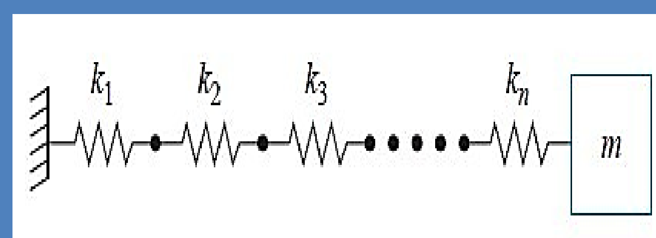
$$\frac{f}{k_{eq}} = \frac{f}{k_1} + \frac{f}{k_2} + \frac{f}{k_3}$$

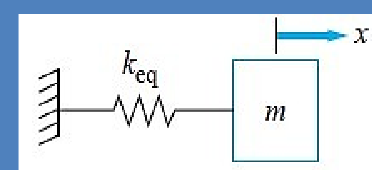
$f = constant \quad \rightarrow (2)$

Hence,

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$k_{eq} = 1/\sum_{i=1}^{n}\left(\frac{1}{k_i}\right)$$





Equivalent springs (Combining stiffness)

▪Springs in parallel l

:

$f_{eq} = f_1 + f_2 + f_3$ ------ (1)

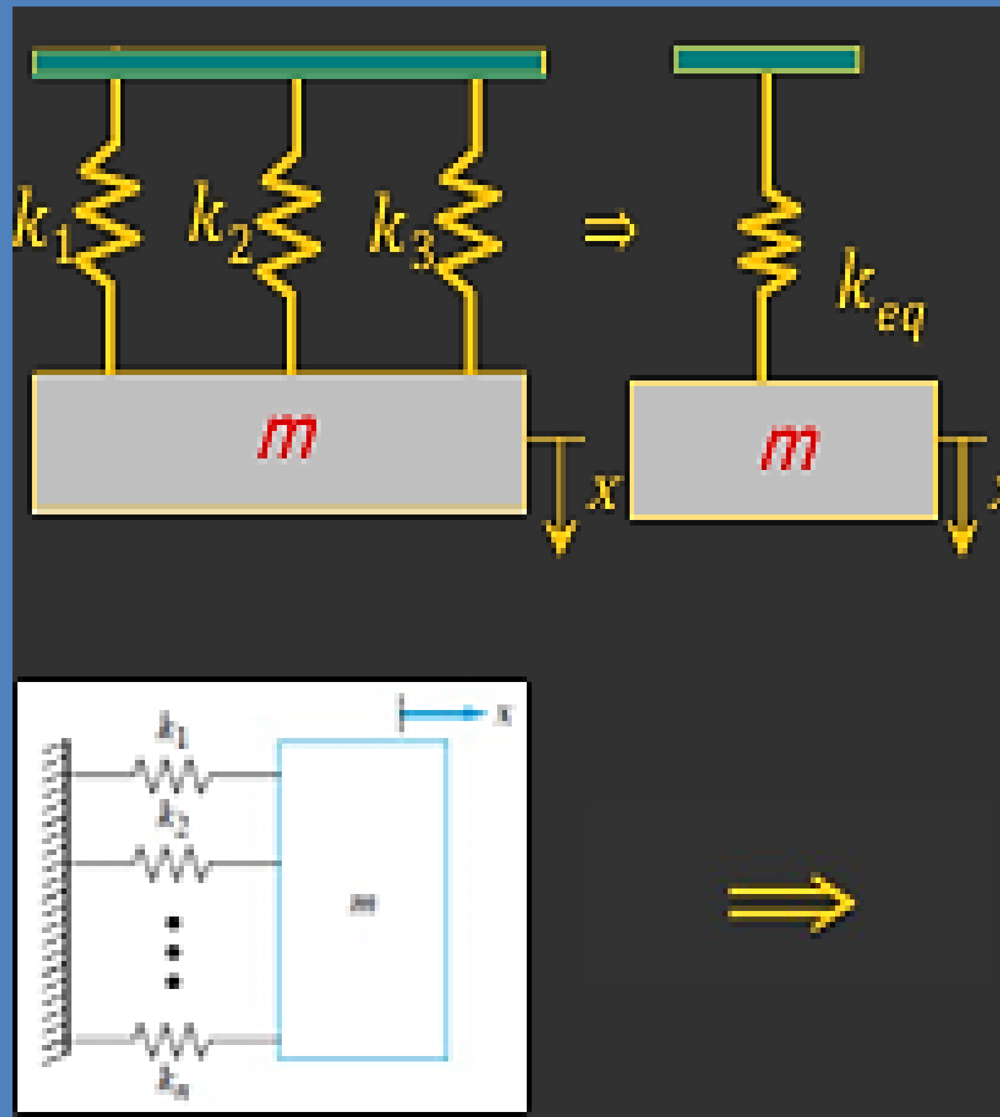
Since, $f\,t = k\,x$

$K_{xeq}\, x_{eq} = K_1 X_1 + K_2 X_2 + K_3 X_3$

$X_{eq} = X_1 = X_2 = X_3$

$K_{eq} = K_1 + K_2 + K_3$

$$K_{eq} = \sum_{i=1}^{n} Kt$$



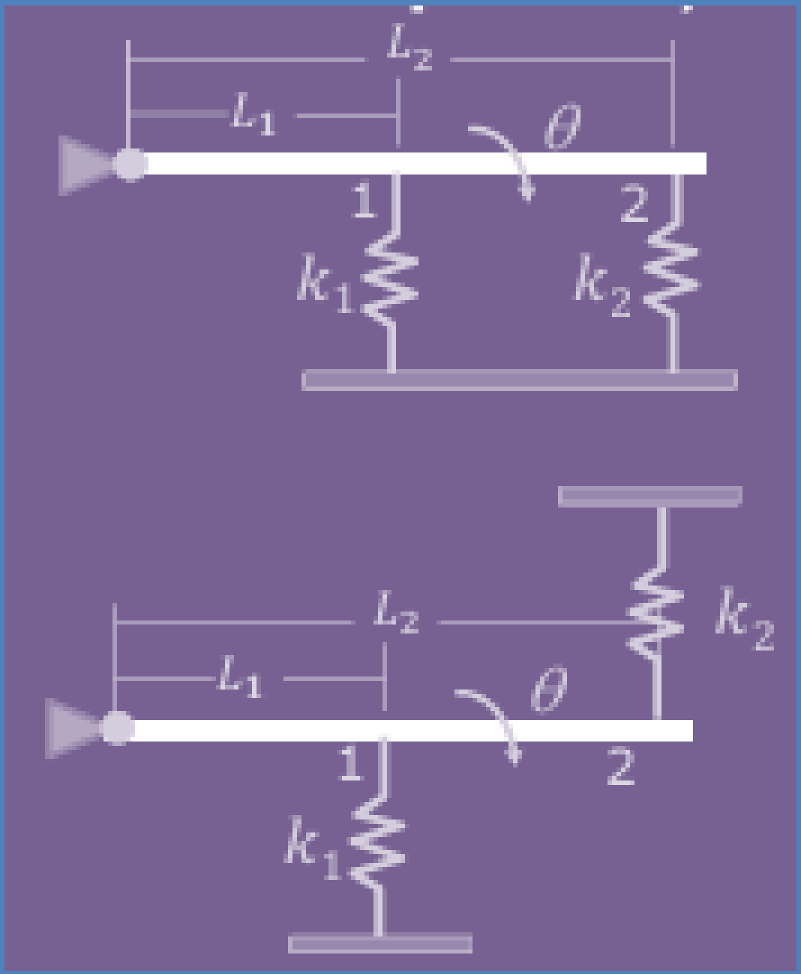
Equivalent springs (Combining stiffness)

▪Combined Springs (not in series nor in parallel):

$$P.E._{eq} = \sum_{i=1}^{n} P.E_i$$

$$P.E_{eq} = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$

$$\frac{1}{2}k_{eq} x_{eq}^2 = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$



Equivalent springs (Combining stiffness)

- Combined Springs (not in series nor in parallel):
- If equivalent spring at point (1)

$$x_{eq} = x_1$$

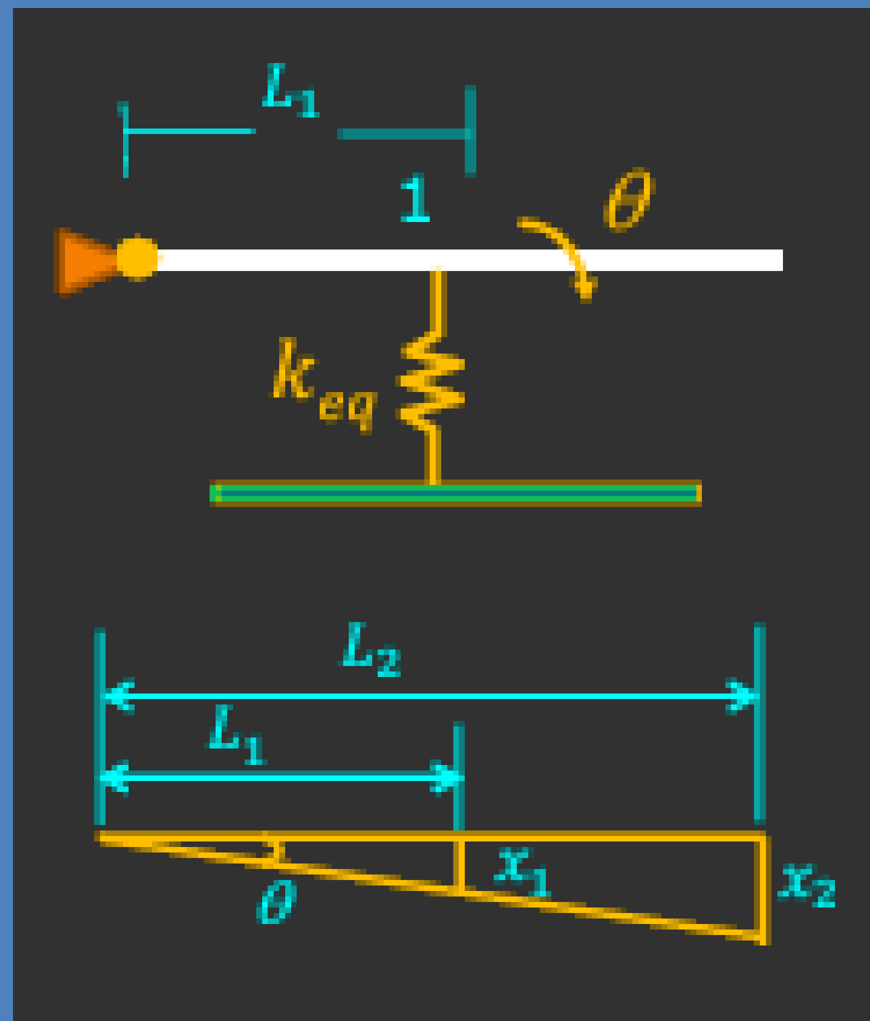
$$\frac{1}{2} k_{eq}\, x_{eq}^2 = \frac{1}{2} k_1\, x_1^2 + \frac{1}{2} k_2\, x_2^2$$

$$k_{eq} = k_1 + k_2 \frac{x_2^2}{x_1^2}$$

$$x = L\,\theta$$

$$Hence\; x_1 = L_1\,\theta \quad \& \quad x_2 = L_2\,\theta$$

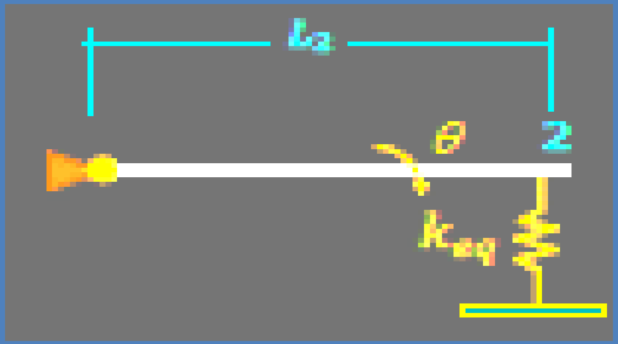
$$k_{eq} = k_1 + k_2 \left(\frac{L_2}{L_1}\right)^2$$



Equivalent springs (Combining stiffness)

- Combined Springs (not in series nor in parallel):
- If equivalent spring at point (1)

$X_{eq} = X_2$



$$\frac{1}{2} K_{eq} X^2_{eq} = \frac{1}{2} K_1 X^2_1 = \frac{1}{2} K_2 X^2_2$$

$$Keq = K1 \frac{X1^2}{X2^2} + K2$$

$$Keq = K1 \frac{L1^2}{L2^2} + K2$$

Equivalent springs (Combining stiffness)

Another method (Using Newton 2nd law for rotational system

(Since, link has negligible mass, then $J = 0$)

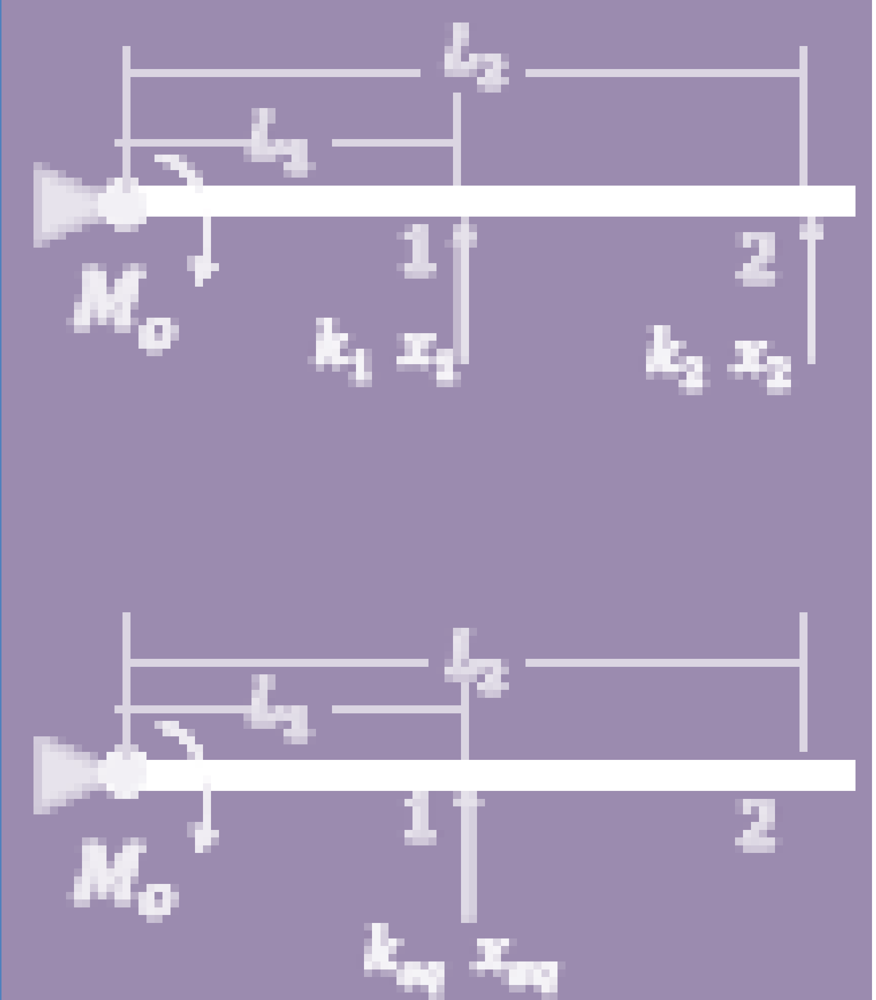
$0 = (k_1\, x_1)L_1 + \; k_2\, x_2\, L_2$ **(1)**

- The equivalent spring at the position of spring (1)

$J\,\theta = \sum M_o$ $\quad 0 = (k_{eq}\, x_{eq})L_1$ **(2)**

At position (1) $x_{eq} = x_1$, **eq. (1) =eq. (2)**

$(k_{eq}\, x_1)L_1 = (k_1\, x_1)L_1 + \; k_2\, x_2\, L_2$

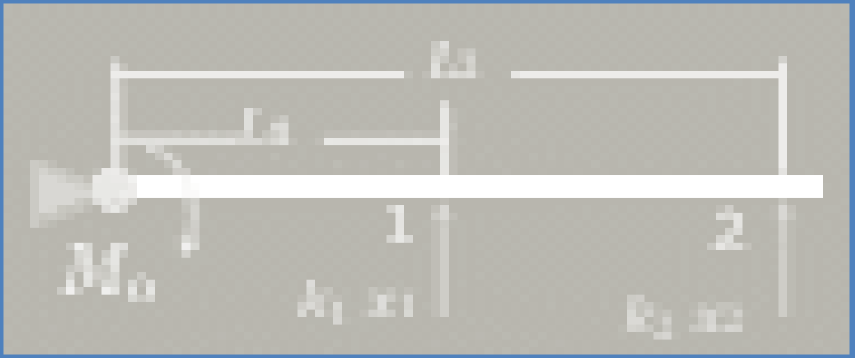


$$Keq = = K_1 + K_2\left(\frac{X1\ l1}{X2L2}\right) = K_1 + K_2\left(\frac{l1}{L2}\right)^2$$

Equivalent springs (Combining stiffness)

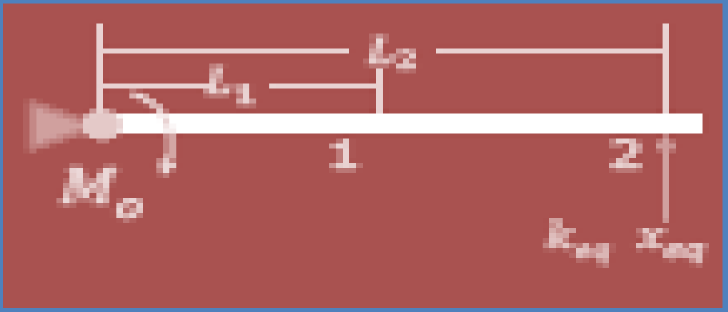
- **The equivalent spring at the position of spring (2)**

$$0 = (k_{eq}\ x_{eq}\ )L2$$



At position (2) X e q = $x2$

$$(k_{eq}\ x_2)L_2 = (k_1\ x_1)L_1 + k_2\ x_2\ L_2$$



$X_1 = L_1\,\Theta$, $X_2 = L_2\,\Theta$

$$K_{eq} = = K1\left(\frac{X1\ l1}{X2L2}\right) + K_2 = K_1\left(\frac{l1}{L2}\right)^2 + K2$$

Equivalent dampers

- **Dampers in series :** $y_{eq} = y_1 + y_2 + y_3$

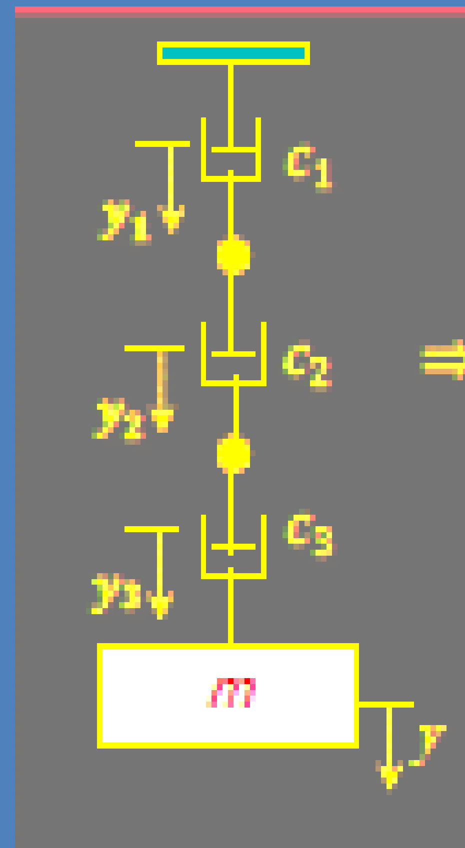
Since, $f(t) = c\dot{y}$

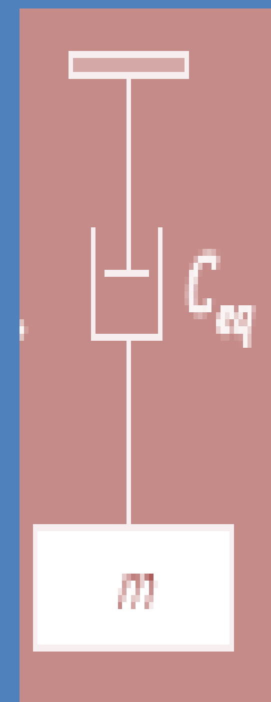
$$\frac{f}{Ceq} = \frac{f}{C1} + \frac{f}{C2} + \frac{f}{C3}$$

f is constant

$$\frac{1}{Ceq} = \frac{1}{C1} + \frac{1}{C2} + \frac{1}{C3}$$

$$Ceq = \sum_{i=1}^{n} \frac{1}{Ci}$$





Equivalent dampers

- **Dampers in parallel :**

$f_{eq}=f_1+f_2+f_3$ --------- (1)

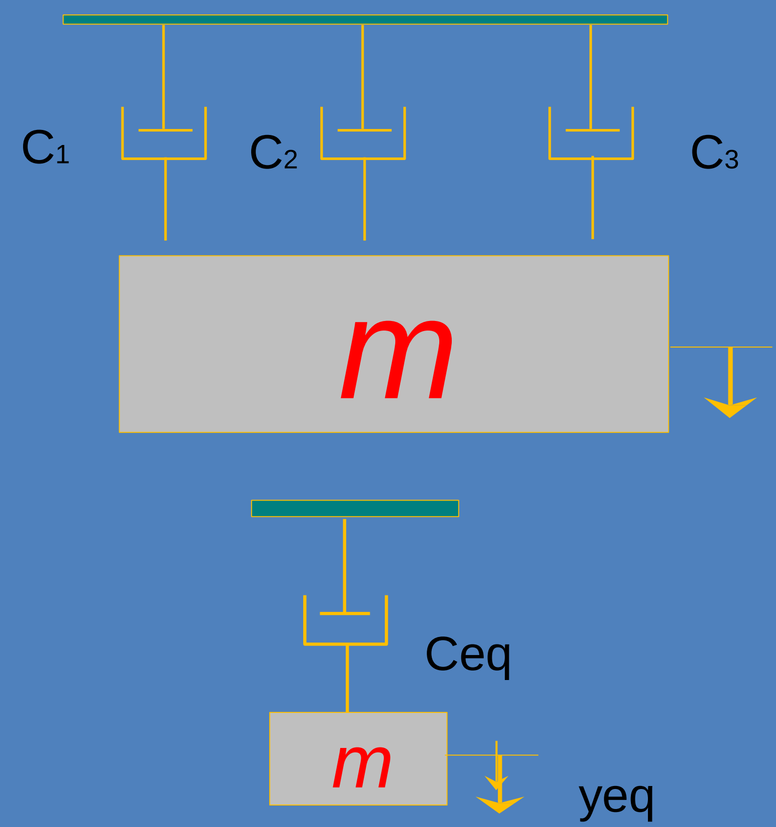
Since f(t) = C $\dot{y}$

$y_{eq}= y_1 + y_2 + y_3$ -------(2)

Since $y_{eq} = y_1 = y_2 = y_3$

Hence $C_{eq} = C_1 + C_2 + C_3$

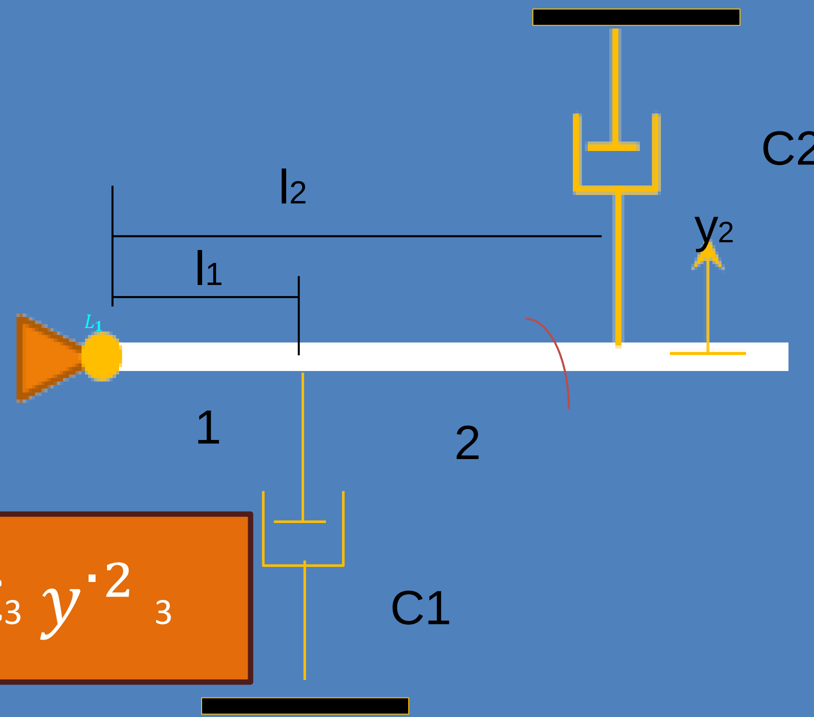
$$Ceq = \sum_{i=1}^{n} Ci$$



Equivalent dampers

Combined dampers (not in series nor in parallel

$$D.E = \sum_{i=1}^{n} D.Ei$$



D.Eeq = $= \frac{1}{2} C_1 y^{\cdot 2}{}_1 + \frac{1}{2} C_2 y^{\cdot 2}{}_2 + \frac{1}{2} C_3 y^{\cdot 2}{}_3$

$\frac{1}{2} C_{eq} y^2 \text{ eq} = \frac{1}{2} C_1 y^{\cdot 2}{}_1 + \frac{1}{2} C_1 y^{\cdot 2}\ 2$

Equivalent dampers

- If equivalent damper at point (1)

$$\frac{1}{2} C_{eq}\, y^2\, eq = \frac{1}{2} C_1\, y^{\cdot 2}{}_1 \; + \frac{1}{2} C_1\, y^{\cdot 2}\, 2$$

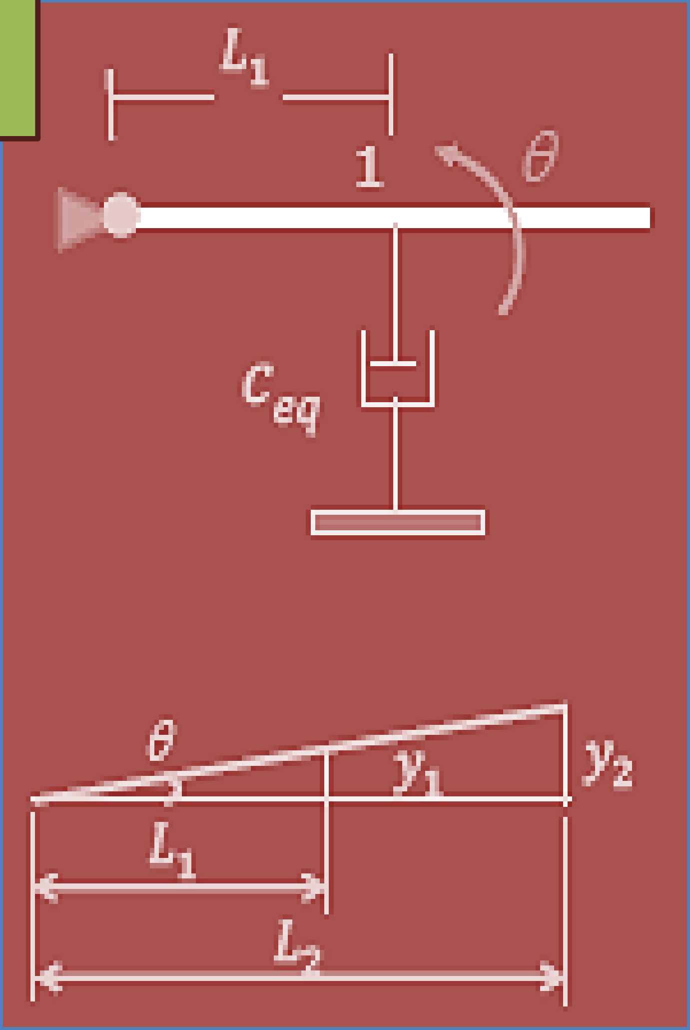
$$y_{eq} = y_1 \qquad \text{then} \quad y^{\cdot}{}_{eq} = y^{\cdot}{}_1$$

$$C_{eq} = C_1 + C_2 \frac{y^{\cdot}2^2}{y^{\cdot}1^2}$$

$y_1 = l_1\, \theta$, then $y_{eq}^{\cdot} = l_1\, \theta^{\cdot}$

$y^{\cdot}{}_1 = l\, \theta^{\cdot}1$, and $y^{\cdot}2 = l\, \theta^{\cdot}2$

$$C_{eq} = C_1 + C_2 \frac{l2^2}{l1^2}$$



Equivalent dampers

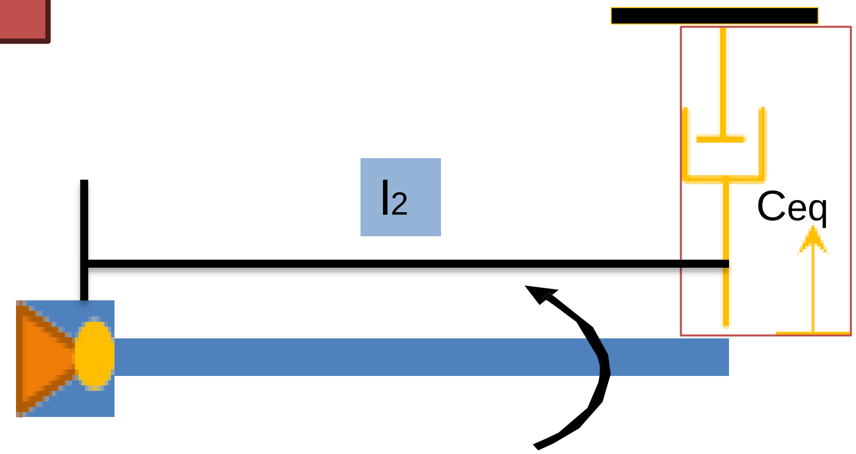
- **Combined damper (not in series nor in parallel):**

- **If equivalent damper at point (2)**

$y_{eq} = y_2$ **then** $\dot{y}_{eq} = \dot{y}_2$

$$\frac{1}{2} C_{eq}\, y^2 = \frac{1}{2} c_1\, y_1^2 + \frac{1}{2} c_2\, y_2^2$$

$$C_{eq} = C_1 \left(\frac{y1}{y2}\right)^2 + C_2 = C_1 \left(\frac{l\,1}{l2}\right)^2 + C_2$$



Equivalent dampers

- **Another method (Using Newton 2^{nd} law for rotational system)**

$J\ddot{\theta} = \sum Mo$

(Link has negligible mass, then J=0)

The equivalent damper at the point (1)

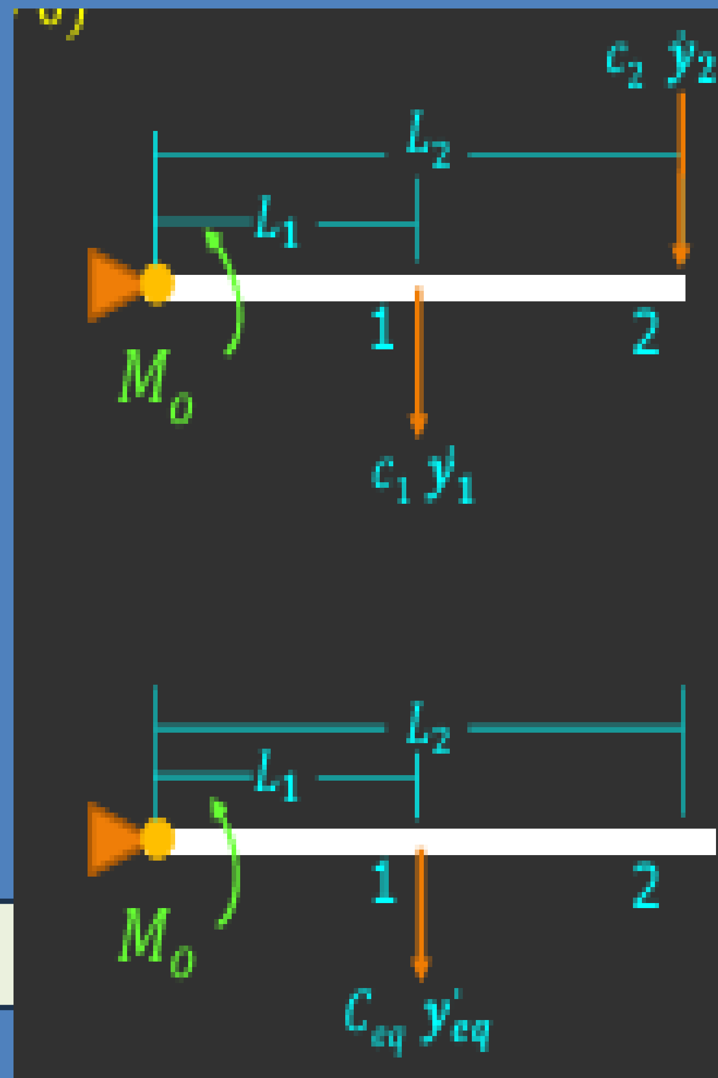
$$0 = (c_1 \dot{y}_1)L_1 + c_2 \dot{y}_2 L_2 \qquad (1)$$

At position (1) $y_{eq} = y_1$

$$J\ddot{\theta} = \sum M \qquad 0 = (c\, \dot{y}1_{eq})\, L\, 1 \qquad (2)$$

$$c_{eq}\, \dot{y}_1 eq\, L_1 = (c_1 \dot{y}_1)L_1 + c_2 \dot{y}_2 L_2$$

$$C_{eq} = C_1 + C2\, \frac{l2^2}{l1^2}$$



Equivalent dampers

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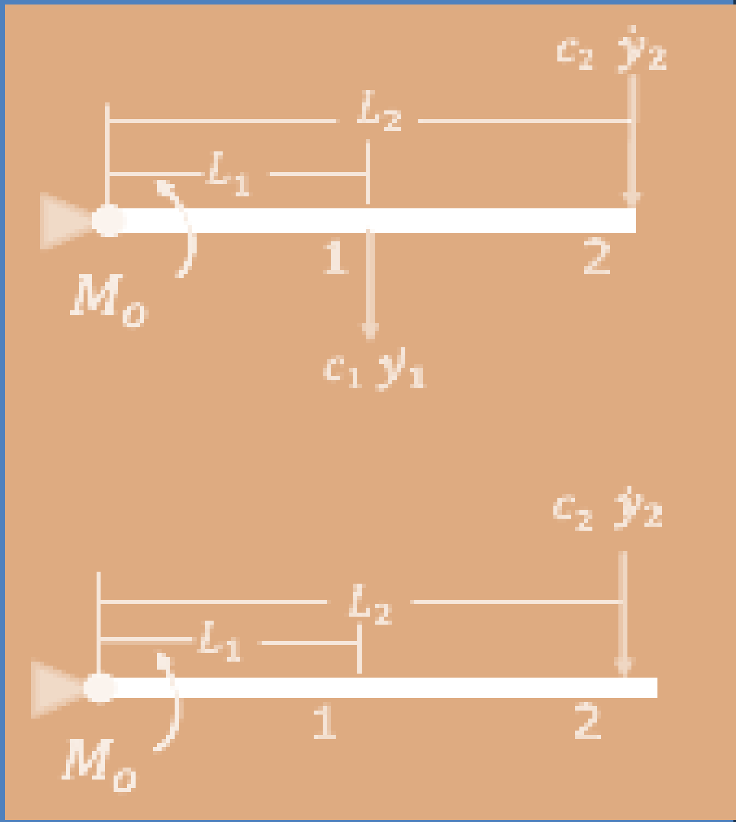

The equivalent damper at the point (2)

$0 = (c_1\ \dot{y}_1)L_1 + c_2\ \dot{y}_2 L_2$ (1)

At position (1) $y_{eq} = y1$

$J\ \theta = \sum M$ $\quad 0 = (c_{eq}\ \dot{y}_{2\ eq})\ L_2$ (2)

$c_{eq}\ \dot{y}_2 eq\ L_2 = (c_1\ \dot{y}_1)L_1 + c_2\ \dot{y}_2 L_2$

$C_{eq} = C_1\ (\frac{y1}{y2})^2 + C_2 = C_1\ (\frac{l1}{l2})^2 + C_2$

Thanks